ASISC MOCK EXAMINATION
MATHEMATICS

Class: XII **Marks: 80**
Time: 3Hrs

Attempt all questions from SECTION A and
all questions <u>EITHER</u> from SECTION B or SECTION C.

SECTION A- 65 MARKS

Question 1:

In subparts (i) to (x) choose the correct options and in subparts (xi) to (xv), answer the questions as instructed.

(i). The value of k if: **[1]**

$$2\begin{bmatrix}1 & 2 & 3\\ -1 & -3 & 2\end{bmatrix} + k\begin{bmatrix}1 & 0 & 2\\ 3 & 4 & 5\end{bmatrix} = \begin{bmatrix}4 & 4 & 10\\ 4 & 2 & 14\end{bmatrix}$$

(a) k=1 (c) k=0

(b) k=2 (d) none of these

(ii) $\int \frac{e^x(1+x)}{cos^2(xe^x)}$ dx equals: **[1]**

(a) -cot(xe^x) + C (c) cot (e^x) + C

(b) tan (xe^x) + C (d) - tan (xe^x) +C

(iii) Value of $\tan^{-1}\left(tan\frac{9\Pi}{8}\right)$ is : **[1]**

(a) $\frac{9\Pi}{8}$ (b) - $\frac{9\Pi}{8}$ (c) - $\frac{\Pi}{8}$ (d) $\frac{\Pi}{8}$

(iv) In the following question, a statement of Assertion is followed by a statement of Reason. Choose the correct answer out of the following choices. **[1]**

Assertion: The sum of the order and the degree of the differential equation $\frac{d}{dx}\left[\left(\frac{d^2y}{dx^2}\right)^4\right]$ is 4.

Reason: The number of arbitrary constants in the particular solution of a differential equation of a third order is zero.

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true, but Reason is not the correct explanation of Assertion.

(c) Assertion is true but Reason is false.

(d) Assertion is false but Reason is true.

(v) If P(A) = $\frac{4}{5}$ and P$(A \cap B) = \frac{7}{10}$, then P(B|A) is equal to: **[1]**

(a) $\frac{1}{10}$ (b) $\frac{1}{8}$ (c) $\frac{7}{8}$ (d) $\frac{17}{20}$

(vi) The value of the determinant $\begin{vmatrix}42 & 1 & 6\\ 28 & 7 & 4\\ 14 & 3 & 2\end{vmatrix}$ is: **[1]**

(a) 0 (c) 12

(b) 1 (d) none of these

(vii) **Statement 1:** The function f(x) = $\frac{4-x^2}{4x-x^3}$ is continuous.

Statement 2: The function f(x) = $\frac{4-x^2}{4x-x^3}$ is discontinuous at exactly three points.

Which one of the following options is correct? **[1]**

(a) Both the statements are true. (c) Statement 1 is true and Statement 2 is false.

(b) Both the statements are false. (d) Statement 1 is false and Statement 2 is true.

(viii) if y = $\sqrt{sin\,x + y}$, then $\frac{dy}{dx}$ is: **[1]**

(a) $\frac{-\cos x}{2y+1}$ (c) $\frac{\cos x}{2y-1}$

(b) $\frac{co}{2y+1}$ (d) none of these

(ix) **Statement 1:** The function f: N → N given by f(x)=2x+3 is one to one and onto.
Statement 2: The function f: R → R given by f(x)=2x-3 is one to one and onto.
Which one of the following options is correct? **[1]**

(a) Both the statements are true.
(b) Both the statements are false.
(c) Statement 1 is true and Statement 2 is false.
(d) Statement 1 is false and Statement 2 is true.

(x) Equality of two matrices. **[1]**

Assertion: If $\begin{bmatrix} 2 & x \\ z & y \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 5 & 3 \end{bmatrix}$ then x= -1, y =3 and z = 5.

Reason: Two matrices are equal if they are of the same order and their corresponding elements are equal.

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) Assertion is true and Reason is false.
(d) Assertion is false and Reason is true.

(xi) If A and B are two independent events, then the probability of occurrence of at least one of A and B is given by: **[1]**

(a) 1 – P(A`) P(B`)
(b) 1 – P(A) P(B`)
(c) 1 – P(A`) P(B)
(d) 1 – P(A) P(B)

(xii) If $B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$, then find B^{-1} **[1]**

(xiii) Show that the function f: R → R given by f(x)=cos x is neither one-one nor onto. **[1]**

(xiv) Is g = {(1,1), (2,3), (3,5), (4,7)} a function? If g is described by g(x) = αx + β, then what values should be assigned to α and β? **[1]**

(xv) Three cards are drawn at random from a pack of 52 cards. What is the probability that the drawn cards are all queens? **[1]**

Question 2: **[2]**

(i) If y= log tan $\frac{x}{2}$, find $\frac{dy}{dx}$.

OR

(ii) The edge of a variable cube is increasing at the rate of 10 cm/sec. How fast is the volume of the cube increasing when the edge is 5cm long?

Question 3: **[2]**

Evaluate: $\int_0^a \frac{\sqrt{x}\,dx}{\sqrt{x}+\sqrt{a-x}}dx$

Question 4: **[2]**

The equation of tangent at (2, 3) on the curve $y^2 = px^3+q$ is y = 4x-7 Find the values of 'p' and 'q'.

Question 5: **[2]**

(i) Evaluate: $\int \frac{sin(x-a)}{sin\,x}dx$

OR

(ii) Evaluate: $\int \frac{sin\,x\,cos\,x}{sin^4\,x+cos^4\,x}dx$

Question 6: **[2]**

Consider function f: R^+ → [-9, ∞] given by $f(x)=5x^2+6x-9$. Prove that f(x) is one-one onto.

Question 7: **[4]**

If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi$, prove that $x^2 - y^2 - z^2 + 2yz\sqrt{1-x^2} = 0$

Question 8: **[4]**

Evaluate: $\int \frac{x}{x^4+x^2+1} dx$

Question 9: **[4]**

(i) The two sides of an isosceles triangle with fixed base 'a' are decreasing at the rate of 3cm/ sec. How fast is area decreasing when the two sides are equal to the base?

OR

(ii) If $f(x) = \begin{cases} |x|+3 & if \quad x \leq -3 \\ 2x & if \quad -3 < x < 3 \\ 6x+2 & if \quad x \geq 3 \end{cases}$

Discuss continuity at x= -3 and x=3.

Question 10: **[4]**

(i) The Kona Bulb Factory produces electric bulbs. It has three machines E_1, E_2 and E_3 which produce 50%, 25% and 25% respectively, of the total daily output of electric bulbs. If it is known that 4% of the bulbs produced by each of the machines E_1 and E_2 are defective. Also, 5% of the bulbs produced by machine E_3 are defective.

(a) Find probability of bulbs produced by machine E_1.

(b) What is probability of choosing a defective bulb from a day's production at the Kona Bulb Factory?

(c) A defective bulb is chosen, what is probability that it was produced by machine E_1?

(d) A defective bulb is chosen, what is probability that it was produced by machine E_3?

OR

(ii) During the annual health check-up at Hamirpur Village doctors were examining the chest X-ray to detect about the TB. In this village 1 in 1000 people suffer from TB. The probability that TB is detected when a person is actually suffering is 0.99. The probability of a healthy person diagnosed to have TB is 0.001.

(a) What is the probability that a healthy person is not detected to have TB?

(b) Find the probability of person not having TB?

(c) What is the probability that a person is diagnosed to have TB?

(d) A person is selected at random in Hamirpur Village and is diagnosed to have TB. What is the probability that he actually has TB?

Question 11: **[6]**

If $A = \begin{bmatrix} 1 & 1 & -1 \\ 3 & 1 & -2 \\ 1 & -1 & -1 \end{bmatrix}$, find A^{-1}.

Using A^{-1}, solve the following system of equations x+ y- z=1, 3x +y+-2z = 3 and x- y-z= -1.

Question 12: **[6]**

(i) Solve the differential equation: (y+ log x) dx – x dy =0, given that y=0, x=1.

OR

(ii) Evaluate: $\int \frac{sin^{-1} x}{(1-x^2)^{3/2}} dx$

Question 13: **[6]**

(i) The owner of a school wants to lay out a running track of 400 metres enclosing a football field the shape of which is rectangular with a semicircle at each end.

(a) If x and y did not the length and breadth of the rectangular region then find the relation between the variables.

(b) What is area of the rectangular region A expressed as a function of x?

(c) What is the maximum value of area A?

(d) What is the area generated if the area of the whole floor is maximised?

OR

(ii) A Publisher wants to publish a book consisting of 700 pages. Each printed page is to have a total area of 80 square centimetre with a margin of 1 cm at the top end on each side and a margin of 1.5 cm at the bottom

(a) If x and y denote the length and breadth of the page and if printed area of the page is A, then determine the relation between A and x.

(b) Find the length of the page for which the printed area is maximum

(c) What is the maximum printed area of the page

(d) What should be the dimension of the printed page of the book?

Question 14: **[6]**

Two numbers electorate random without replacement from the first six integers

(a) If X denotes the larger of two numbers, then determine X

(b) What is P (3)?

(c) Find probability distribution of X.

(d) Find E(X).

SECTION B- 15 MARKS

Question 15 **[5]**

In subparts (i) and (ii), choose the correct options and in subparts(iii) to (v), answer the questions as instructed.

(i) If $\vec{a} = x(\hat{\imath} + \hat{\jmath} + \hat{k})$ is a unit vector, then:

(1) x= $\pm\frac{1}{\sqrt{3}}$

(2) x= $\frac{1}{\sqrt{3}}$

(3) $|\vec{a}| = 1$

(4) $|\vec{a}| = \pm 1$

(a) only (1)

(b) (1) and (3)

(c) (2) and (3)

(d) (1) and (4)

(ii) The value of λ for which the vectors $3\hat{\imath} - 6\hat{\jmath} + \hat{k}$ and $2\hat{\imath} - 4\hat{\jmath} + \lambda\hat{k}$ are parallel is:

(a) $\frac{2}{3}$ (a) $\frac{3}{2}$ (a) $\frac{5}{2}$ (a) $\frac{2}{5}$

(iii) If the direction cosines of line are k, k, k then find the value of k.

(iv) A plane passes through points (2,0,0), (0,3,0) and (0,0,4). Find the equation of the plane.

(v) Write down a unit vector, making an angle of 30^0 with the positive direction of X- axis.

Question 16: **[2]**

(i) If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = 4$, $|\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 6$, find angle between $\vec{a}$ and $\vec{b}$.

(ii) If $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = -8$, find $|\vec{a} + 3\vec{b}|$.

Question 17: **[4]**

(i) Find the equation of the plane parallel to the plane 2x-3y-5z + 1=0 and distance 5 units from the point (-1,3,1).

OR

(ii) Find the foot of the perpendicular drawn from the point $2\hat{\imath} - \hat{\jmath} + 5\hat{k}$ to the line $\vec{r} = (11\hat{\imath} - 2\hat{\jmath} - 8\hat{k}) + \lambda((10\hat{\imath} - 4\hat{\jmath} - 11\hat{k})$.

Question 18: **[4]**

Find the area of the region enclosed by the parabola $y^2 = 4ax$ and the line $y = mx$.

SECTION C- 15 MARKS

Question 19 **[5]**

In subparts (i) and (ii), choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(i) If $C(x) = \frac{1}{3}x^3 + x^2 - 8x + 5$, then the marginal cost is:

(a) $\frac{1}{2}x^2 + x - 8$ (c) $x+2 - \frac{8}{x}$

(b) $x^2 + 2x - 8$ (d) none of these

(ii) Given $b_{yx} = 0.8$ and $b_{xy} = 0.45$. Choose the correct option.

(1) r = 0.6 (3) $r = \mp\sqrt{b_{yx}b_{xy}}$

(2) r = 0.6 (4) sign of r, b_{yx} and b_{xy} should be same

(a) (1) and (2) (c) (2), (3) and (4)

(b) (1), (2) and (4) (d) (3) and (4)

(iii) What is the relation between AC and MC?

(iv) If two regression lines are 8x- 10y+66=0 and 40x-18y =214, then find the mean value of x and y.

(v) If demand function of a monopolist is given by $p=1500-2x^2-x^2$. Find the marginal revenue.

Question 20: **[2]**

(i) The total cost function for x units is given by $C(x) = \sqrt{6x + 5} + 2500$. Show that the marginal cost decreases as the output x increases.

OR

(ii) The manufacturing cost of an item consists of ₹ 900 as overheads, the material cost ₹ 3 per item and the labour cost ₹$\frac{x^2}{100}$ for x items produced. How many items must be produced to have average cost minimum?

Question 21: **[4]**

If the two regression lines 4x-5y+33=0 and 20x-9y-107=0, calculate:

(a) The arithmetic means of x and y respectively.

(b) Estimate the value of x when y=7.

(c) Find the variance of y when $\sigma_x = 3$.

Question 22: **[4]**

(i) A housewife wishes to mix together two kinds of food, x and y, in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin content of one kg of food given as follows:

	Vitamin A	Vitamin B	Vitamin C
Food X	1	2	3
Food Y	2	2	1

One kg of food X cost ₹6 and one kg of food Y cost ₹ 10. Find the least cost of the mixture which will produce the diet.

OR

(ii) Solve the following problem graphically:

Minimize and Maximize $Z=3x+9y$ subject to the constraints:

$x+3y \leq 60$

$x+y \geq 10$

$x \leq y$

$x \geq 0, y \geq 0$

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